Optimal Timing of Review Meetings: Periodicity, Frequency and the Value of Waiting

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Abstract

In this paper we study the optimal timing of management meetings at which risk management (hedging) decisions are taken with respect to a real options position. Unlike earlier research, which typically exogenously assumes a periodic meeting schedule, our model allows for flexible (non periodic) scheduling of meetings. We show that if unit meeting costs are constant (no matter whether they are scheduled in a periodic fashion or not), periodicity is unlikely to be optimal. However, when we relax the cost constancy condition we are able to derive parametrically a critical cost frontier above which the net returns from periodicity always dominate. That is, the issue of whether periodic scheduling of meetings is optimal is settled via comparison of actual costs to the computed critical cost bound. At first sight, our results may seem to be consistent with the simple intuition that as risk increases, more meetings should be set, and, eventually, scheduling policy be updated from a periodic to a flexible basis. However, working within an optimization framework we are able to show that such seemingly intuitive claims are of restricted validity. We show that, for certain ranges of parameter values, increasing risk does indeed necessitate a shift from periodic to flexible scheduling, but critically, simultaneously the optimal response is to reduce meeting frequency, as compared to the previous periodic regime. Thus, our model contributes to an understanding of how rich the set of potential meeting policies needs to be in order to obtain optimality.

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1 INTRODUCTION

When risk exposure is generated by the holding of real assets, rather than simple financial claims, the method of real options can be applied to aid decision makers take optimal decisions. In this approach valuation techniques are developed which rely heavily upon correspondences that exist between the payoff profiles of financial and real options. While recognizing the importance of such correspondences, there also exists important differences. For instance, Luehrman (p52, (1998)) has argued that, "most business opportunities are unique, so the likelihood of finding a similar option is low". This means that the development of programmed trading rules for real options exposure is unlikely to be economically justifiable, since, given the complex nature of payoff interactions and the non repeatable nature of the claims profile, the development cost could in many cases be prohibitive. Thus, in practice, decision makers meet to exchange information and try to form a measured consensus on optimal policy. In order to capture this real world feature of the significance of such meetings, this research considers how an organization can optimally choose the style of scheduling for managerial meetings in order to manage real options exposure. We note that by style we mean both overall frequency of meetings during a given cycle and inter-meeting time which for obvious reasons we refer to as the (management) accounting period. This may be categorized as periodic (constant timing between scheduled meetings), or flexible (variable timing between scheduled meetings).

Potentially, a large number of differing factors may impinge upon the afore-mentioned timing issue. However, in order to develop a clear understanding of at least one of the principal determinants, our focus is restricted to consideration of a hedging decision[1] for a specialized input. When input prices are stochastic, it is well known that decision makers cannot simply react by replacing deterministic parameter values with expectations over the parameters. The extensive literature[2] on the value of waiting has demonstrated why decision rules have to be modified to explicitly incorporate the effect of the higher moments of the decision-relevant characterizing distributions. The thrust of this stream of real options research is to show why waiting / delaying decisions may be of considerable value; it is thus of some importance to ask how management accounting procedural rules designed to facilitate optimal decision making, incorporate these considerations. At this point it is perhaps worth stressing that a direct way of incorporating value of waiting considerations is to allow the length of accounting periods to be freely / flexibly set. Thus, on a priori grounds, we suggest that it is not obvious that management accounting periods should be chosen to be periodic[3].

[1]The principle feature of our model that makes our results innovative is the meetings optimization which overlays the formalized dynamic structure of the decision problem. The hedging decision upon which we focus is just one implementable example of where such dynamics arise in practice. In principle, the general techniques we develop here could be applied to a wide class of decision problems.

[2]See for instance Dixit and Pindyck (1994).

[3]It is perhaps worth reiterating that by accounting period we mean the length of time between management meetings. Typically this may differ from the frequency with which management accounting data is compiled. For instance, direct materials variances may be recorded on an hourly basis in the management accounting system. However, management might meet only on a monthly basis to evaluate the aggregate results. Our concern is with the frequency with which meetings are convened, rather than the frequency with which raw

The paper is organized as follows. In the following sub-section we review related literature. In Section 2 we develop the simplest version of our generic hedging model in which management have to decide on input purchases in an inflationary price environment. At the commencement of each accounting period, management meet to review risk exposure. In particular we focus upon how input price risk can be controlled by the signing of advance purchase agreements[4]. The agreements allow management to order the input in advance at the existing price, provided they commit to take delivery of a certain agreed quantity as specified in a formal supplier contract. Commitment to an advance purchase agreement is assumed potentially beneficial because the input price evolves stochastically with positive drift. Even though the input price process has positive drift, we show why management will not want to commit to purchase all projected requirements at the current prevailing price[5]. The intuition here is that scheduling a follow-up meeting to review the purchasing agreement after observing how prices have subsequently evolved has value, or expressed more succinctly, there is positive value of waiting. Our results in Section 2 concerning optimal (waiting) policies are derived under the assumption that one follow-up meeting is scheduled to occur at the mid-point of the overall decision cycle[6]. This simplifies the initial analysis and allows us to concentrate upon how the value of waiting arises in the simplest setting. In section 3 we relax this assumption and formulate a model in which management are free to choose optimally where in the chosen decision cycle to locate the one follow-up meeting. This choice determines whether or not it is optimal for the overall decision cycle to be split up into two balanced / periodic accounting periods. We derive the optimality equations for the timing of the meeting which show that, in general, periodicity for meetings will not be optimal.

In section 4 we further generalize the results by introducing the possibility of any (integer) number of meetings and a simple meeting cost function. This allows us to develop an analysis of the optimal number of meetings n^*. Next, we consider the optimality of a periodic meeting strategy if the cost of meeting flexibly is greater than the cost of meeting on a periodic basis. In particular we determine a critical bound for the cost difference which, if satisfied, insures that a periodic meeting strategy dominates a flexible strategy. This result allows us to provide a theoretical justification for why some organizations convene meetings on a periodic basis while others do not. That is, our results suggest that while it can be optimal for a wide class of organizations (for which the cost bound is satisfied) to adopt periodic provision of accounting data to management meetings, such a policy does not have universal prescriptive appeal. Moreover, by adopting a formal optimization approach we are able to show that, although an increase in risk may eventually result in decision makers switching from a periodic to

data is recorded. As this discussion illustrates, one reason to consider first meeting frequency, rather than recording frequency, is because the sum of hourly data processing costs may be far less than the opportunity cost of convening a management meeting to discuss and evaluate the real options position.

[4]We assume that the input is of a somewhat specialized nature, for example a part completed industrial subcomponent for which a secondary re-sale or derivatives market does not exist. For a wider class of situations see also Bowden (1995).

[5]This arises because of the positive value of waiting which we later establish for our model.

[6]We subsequently allow for $n > 1$ follow-up meetings.

a flexible meetings schedule, a side effect of this switching policy can be that the optimal number of managerial meetings is reduced, thus demonstrating the naivety of adopting a heuristic rule based upon the intuition that increasing risk will increase the desirability both of flexibility and of an increase in the number of meetings. We present concluding comments in section 5.

1.1 Related Literature

Since one can view the inter-meeting time as defining the optimal length of an accounting period, it is natural to consider the related accounting literature on conditional investigation policies. The first economics based decision-theoretic study of the frequency of accounting periods[7] was Hughes (1975). He examined optimal information acquisition frequency within a dichotomous state, Markov, production process setting and showed that the optimal timing between reporting need not be uniform (periodic). His approach was closely related to standard, operations management, reliability models[8] which assume a time invariant model setting. However, as Hopp and Nair (p203 (1991)) observe, "the vast majority of these models assume that technology remains constant, so that old machines are replaced with identical models. However, in many important classes of equipment, ..., replacements are made explicitly to take advantage of improved technology. For these applications, the issue is not to determine the economic life of the equipment by comparing increasing maintenance costs with fixed replacement costs. Instead the decision maker must compare the benefits of replacing the current equipment with the most attractive alternative currently on the market versus waiting for a better alternative to become available". That is, for an important class of managerial problems, choice over timing is intimately related to choice of a hedging strategy. The class of problems in which hedging issues arise is clearly larger than just equipment replacement problems. For instance Kahn (1987) develops a model in which hedging strategies are defined with respect to input inventories. We choose also to look at a form of input inventory hedging problem. However, whereas Kahn (1987) assumes decision / meeting periods are at fixed intervals apart, we allow mangement the opportunity to freely choose the meeting interval and, appealing to value of waiting arguments, show that typically it will not be optimal to convene meetings at equal times apart. This feature also differentiates our work from Hansen and Sankar (1998), who look at the frequency of accounting data collection for forecasting purposes, but within a model in which the time intervals are by assumption always of equal relative length. The model closest in spirit to our work is that of Huberman (1988). He develops a queuing theoretic model in which the benefit of an activity increases stochastically as a function of the time since the last activity, and where scheduling an activity (meeting) is costly. Interestingly he is able to derive the optimality of periodic managerial meetings. However, his model setting differs from ours in at least one important respect. Stochasticity is modelled via a (discrete) exponentially distributed random arrival rate for new projects and, hence, once a project is adopted, it has no other impact upon subsequent project decisions. Given our interest in the class of input

[7]For a more extensive review of the early literature see Magee (1986).

[8]See McCall (1965) and Sherif and Smith (1981) for reviews.

price hedging problem, we choose instead to model the stochastic input price as a continuous variable through time and, critically, assume that sequential advance purchasing decisions in one period can affect subsequent returns. This sequential dependendence explains why waiting / hedging considerations arise in our model which are not present in the Huberman setting.

2 THE SIMPLEST PERIODIC MEETING MODEL

The description of the model will commence by outlining a simple, albeit generic, decision problem faced by management, in which advance purchasing agreements for an input may be made in order to hedge against the possibility of input price rises. It is assumed that inventory holding costs are not an issue since the advance purchasing strategy simply involves agreeing to fix the price of future timely deliveries. That is, the strategy does not involve taking delivery of physical stock in advance[9]. Initially, attention will be restricted to a simple periodic / balanced accounting regime in which one follow-up meeting occurs at the mid-point of the overall decision cycle.

2.1 The Balanced Two Meeting Problem

Since our main concern is with identifying departures from periodicity, we are principally interested in the relative length of time between meetings rather than actual clock time. Hence we shall assume throughout that the decision cycle under consideration is normalized to be of unit length and one should note this has implications for interpreting magnitudes for 'realistic' parameter values such as for input price volatility.

In order to denote the timing of meetings, we introduce the convention that the first meeting at the commencement of a decision cycle is always time labelled as t_0 and in the general case of an n meeting policy the end of the decision cycle is time labelled t_n. Given the normalization we assume $t_0 = 0$ and $t_n = 1$.

In this subsection we make the simplest assumption[10] that two managerial decision meetings occur which divide the overall decision cycle into two periods of equal length. That is, the first meeting occurs at the beginning of the cycle at $t_0 = 0$ and the second in the middle at $t_1 = 0.5$ and the cycle ends at $t_2 = 1$. We initially impose periodicity in order to focus first upon characteristics of the generic hedging decision in the simplist setting (recall hedges have potential value, because there is an expectation that prices will rise). The time order diagram reflecting this decision regime is as follows:

Time Order Line

[9] Gietzmann and Ostaszewski (1999)a. show how, if the input needs to be held in stock, inventory carrying costs can be included in the analysis.

[10] We later generalize to allow arbitrary n meetings that can occur at any time.

At $t_0 = 0$:
Observe the current input price b_{t_0}.
Predict $E(b_{t_1})$ the expected price for the following accounting interval, where we assume $E(b_{t_1}) > b_{t_0}$.
Agree with the supplier to fix the price of v_{t_0} units of input at b_{t_0}.

Between $t_0 = 0$ and $t_1 = 0.5$:
Use x_{t_0} units of input in production

At $t_1 = 0.5$:
observe b_{t_1}
agree with the supplier to fix the price of v_{t_1} units of input at b_{t_1}.

Between $t_1 = 0.5$ and $t_2 = 1$:
use x_{t_1} units of input in production.

As the above time order line makes clear, all the subscripts are timings. So we will drop the t subscript when a specific time is being referenced. To simplify further the presentation it is assumed[11] that the functional form for the gross profit returns function is $2\sqrt{x_t}$. Given an initial value for v_0, management need to choose the optimal first period production plan x_0^* such that:

$$x_0^* = \arg\max 2\sqrt{x_0} - b_0 x_0 + F_1(v_0 - x_0, b_0), \tag{1}$$

where $F_1(.)$, is the expected indirect profit function for the subsequent period (which we will derive below) and $v_0 - x_0$ is the hedge. It is convenient to reformulate this production planning problem in terms of the hedge quantity, which shall be denoted u. That is, given $v_0 - x_0 = u$ we can rewrite (1) in terms of choice of u as:

$$u^* = \underset{0 \leq u \leq v_0}{\arg\max}\ 2\sqrt{v_0 - u} - b_0(v_0 - u) + F_1(u, b_0). \tag{2}$$

Let us now turn to consider $F_1(u, b_0)$. We shall derive its form by considering how it is made up over three ranges which depend upon the interaction between u and the input prices. In particular assume that management have just met at time t_1 and are considering how many units to produce, having just observed the current price $b_1 = \tilde{b}_1$, and having previously committed to carrying forward u units of input at price b_0 into the period commencing t_1.

- {First range} Consider the limiting case of $u = 0$. In this case:

$$x_1^* = \left(\frac{1}{\tilde{b}_1}\right)^2,$$

implying the indirect returns function $G(u, b_0)$ in this first region is:

$$G^1(0, b_1 = \tilde{b}_1 | b_0) = \frac{1}{\tilde{b}_1}.$$

[11] In general we require concavity of the returns function. However, in our presentation we specialize to the square root specification since this considerably clarifies the argument.

- {Second range} If $x_1^* \geq u > 0$ we see that:

$$G^2(u, b_1 = \widetilde{b}_1 | b_0) = \frac{1}{\widetilde{b}_1} + (\widetilde{b}_1 - b_0)u.$$

- {Third range} If $u \geq x_1^*$ we have:

$$G^3(u, b_1 = \widetilde{b}_1 | b_0) = 2\sqrt{u} - b_0 u.$$

In the first range at time t_1 management merely match current input purchases to optimal production. For the second range they do likewise; however, having fixed the price of u units of input, if $\widetilde{b}_1 > b_0$, a gain of $(\widetilde{b}_1 - b_0)$ per unit is earned from fixing the price in advance for u units. Alternatively, when $\widetilde{b}_1 < b_0$, the loss per unit is $(\widetilde{b}_1 - b_0)$. In the third range too much stock has been carried forward and so no more is purchased, with all of it being processed since we assume no market for returned input.

The important features of the model are determined by focusing upon the value u takes when the second and third ranges meet, that is when:

$$G^2(u, b_1 = \widetilde{b}_1 | b_0) = \frac{1}{\widetilde{b}_1} + (\widetilde{b}_1 - b_0)u = 2\sqrt{u} - b_0 u = G^3(u, b_1 = \widetilde{b}_1 | b_0), \tag{3}$$

from where we have:

$$u = x_1^*(\widetilde{b}_1) = \left(\frac{1}{\widetilde{b}_1}\right)^2. \tag{4}$$

It is straightforward to show that $G(u, b_0)$ is a piecewise twice-differentiable function of u, with $\widetilde{b}_1$ defining the switching point between regimes. The value of u for this critical $\widetilde{b}_1$ shall be denoted $u(\widetilde{b}_1)$. This implies continuity when passing from G^2 to G^3. Next we note that:

$$F_1(u, b_0) = E[G(u, b_1 | b_0)],$$

where expectations are taken over b_1. Since we shall assume throughout that input prices are expected to rise:

$$b_0 < E(b_1),$$

it will never be optimal to locate in the first range. Thus under the assumption that b_1 is distributed continuously with density function $q(b_1)$ over the range $[\underline{b}_1, \overline{b}_1]$, integration over $G^2(.)$ and $G^3(.)$ gives:

$$F_1(u, b_0) = \int_{\underline{b}_1}^{\widetilde{b}_1} \{\frac{1}{b_1} + b_1 u\} dq(b_1) + \int_{\widetilde{b}_1}^{\overline{b}_1} 2\sqrt{u} dq(b_1) - b_0 u. \tag{5}$$

From (2) the first order condition $\frac{\partial F_1(u,b_1)}{\partial u} = b_0$ subject to (4) being satisfied gives us the necessary condition for a stationary point:

$$\begin{aligned} b_0 &= \int_{\underline{b}_1}^{\widetilde{b}_1} b_1 dq(b_1) + \frac{1}{\sqrt{u}} \int_{\widetilde{b}_1}^{\overline{b}_1} dq(b_1) \\ &= \int_{\underline{b}_1}^{\widetilde{b}_1} b_1 dq(b_1) + \widetilde{b}_1 \int_{\widetilde{b}_1}^{\overline{b}_1} dq(b_1) \\ &\equiv E[b_1 \wedge \widetilde{b}_1], \end{aligned} \tag{6}$$

that is, the right-hand side requires the censoring of the probability density of the input price, so that the expectation of the censored process $\widetilde{b}_1$ equals the initial period price b_0. We shall now briefly consider an intuitive explanation for the nature of the censor.

2.2 Reinterpeting the Censor Within an Options Framework

Let us now reinterpret the optimal advance purchasing strategy within an options framework. We need two conditions to be satisfied. First recalling (6) we require the censor to be chosen so that:

$$b_0 = E[b_1 \wedge \widetilde{b}_1]. \tag{7}$$

If we take the censor to be the strike price on a plain vanilla call option, the payoff from holding x call options is given by:

$$\int_{\widetilde{b}_1}^{\overline{b}_1} (2\sqrt{x} - \widetilde{b}_1 x) dq(b_1) - x \int_{\widetilde{b}_1}^{\overline{b}_1} (b_1 - \widetilde{b}_1) dp(b_1), \tag{8}$$

where the first term is the expected payoff when x options are exercised at a strike of $\widetilde{b}_1$ under the subjective probability measure with density $q(.)$, and the second term is the risk compensation premium for the options under an assumed risk-neutral probability measure with density $p(.)$. Thus the second condition which identifies the optimal number of option contracts x^* is given by the necessary first-order condition:

$$\int_{\widetilde{b}_1}^{\overline{b}_1} (\frac{1}{\sqrt{x^*}} - \widetilde{b}_1) dq(b_1) = \int_{\widetilde{b}_1}^{\overline{b}_1} (b_1 - \widetilde{b}_1) dp(b_1). \tag{9}$$

We wish (7) and (9) to be simultaneously satisfied; following (4) consider choosing the number of call options for purchase to be:

$$x^* = \frac{1}{\widetilde{b}_1^2}. \tag{10}$$

This means that (9) now requires:

$$0 = \int_{\widetilde{b}_1}^{\overline{b}_1} (b_1 - \widetilde{b}_1) dp(b_1) = \int_{\widetilde{b}_1}^{\overline{b}_1} b_1 dp(b_1) - \widetilde{b}_1 \int_{\widetilde{b}_1}^{\overline{b}_1} dp(b_1),$$

which immediately implies

$$E_p(b_1) = E_p[b_1 \wedge \widetilde{b}_1].$$

But by risk-neutrality $b_0 = E_p(b_1)$; we thus arrive at the condition $b_0 = E_p[b_1 \wedge \widetilde{b}_1]$ as being the optimality condition for the choice of strike and hedge quantity. This looks just like (6) and if both conditions hold (eg, at its simplest, if the densities $p(.), q(.)$ were equal[12]) we have a natural interpretation for the censor: as providing both the strike price and, via (10), the optimal number of option contracts to purchase.

Having established the intimacy between the terms of the optimal advance purchase agreement and a call option it is also important to understand differences that exist between the two instruments. Consider the issue of portfolio rebalancing. In our chosen real options context, instantaneous rebalancing might not be possible. Unlike for OTC financial options, markets for idiosyncratic inputs typically do not exist. That is, the organization is not free to rebalance portfolio exposure in arbitrary quantities at any point in time; in particular, there might not be an equivalent risk-neutral density $p(.)$, nonetheless our framework remains valid and prescribes optimal behaviour and may thus be regarded as an extension of the traditional approach. To illustrate this point further, let us consider the example of an advance purchase problem faced by an airline. In part in order to hedge against price uncertainty an airline may enter into an advance purchase agreement for future delivery of n airliners at b_0 per unit. If subsequently the price falls to $b < b_0$, mangement are unlikely to want to increase the order quantity until $b_0 - b$ is a sufficently large amount[13]. Secondly, convening a meeting to "consider all the facts" may be difficult to schedule and, once convened, may involve a lengthy process to attain an agreed outcome[14]. Essentially this arises because, for the type of real (input) option which we are considering, the payoff profile may be related to strategic considerations; for instance, if the output market has complex interactions these will need to be carefully considered by a team of multi-disciplinary management. That is, whereas a financial option is essentially a free standing instrument and can be analyzed in exclusion from other operating decisions, we consider a class of problem for which an integrated management solution is required[15].

To summarize, the censor in our model can be interpreted as the optimal strike price for a call option that gives the holder the right to purchase an input at the strike at some future date. However, the complexity involved with the real option means that the analogy should not be pushed too far, since organizational transaction (meeting) costs have an important bearing upon

[12]Outside this paragraph we do not insist that a risk-neutral density exists.

[13]In our model we assume that the principal constraint on entering into a new advance purchase agreement is the need to convene another meeting for management to "go over the facts". If, however, meeting costs were relatively small another strategy would be to use conditional investigation in which a meeting was convened whenever $b_0 - b$ rose to a certain cut-off level.

[14]We formally introduce meeting costs in section 4.

[15]Thus, for the same reason, a programmed trading rule such as might be used for hedging plain vanilla financial options is unlikely to perform well for the real option we have in mind. Indeed in the context of a relatively short sequence of idiosyncratic purchases, programming all the dynamic strategic interactions which are constantly changing over time, may be extremely costly and difficult; so abandoning this approach, the alternative is to schedule meetings bringing in the experts together to analyze the facts.

the optimal frequency with which decisions are re-evaluated. Having now analyzed a simple two meeting setting in which meeting scheduling is constrained to be balanced, let us now turn to consider how value of waiting arguments interplay with choice of length of accounting period in such a fashion that non-periodic meetings may be desirable.

3 ALLOWING THE TIMING OF THE SECOND MEETING t_1 TO BE CHOSEN FLEXIBLY

Let us now proceed as before with one meeting at t_0 and another at t_1. However, unlike in the previous section where we restricted consideration to periodic meetings ($t_1 = 0.5$), we shall now relax this constraint and simply replace it by the more general requirement that t_1 fall in the interval

$$0 \leq t_1 \leq 1.$$

We need to derive the generalized returns function $R(t_1)$ that[16] applies in this setting, so that we can solve for the optimal timing t_1^*. We do so in a number of steps. First we specify the diffusion process for the input price variable.

3.1 The Input Price Process

We shall make the common assumption that the input price b is distributed log-normally with instantaneous mean $\overline{\mu}$ and variance $\overline{\sigma}^2$ as:

$$q(b, \overline{\mu}t_1, \overline{\sigma}^2 t_1)$$

where for the unit time interval

$$q(b, \mu, \sigma^2) = \frac{1}{b\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\ln\frac{b_0}{b} - (\mu - \frac{1}{2}\sigma^2)}{\sigma}\right)^2\right\}$$

and for a general time interval of length t_1 we have:

$$q(b, \overline{\mu}t_1, \overline{\sigma}^2 t_1) = \frac{1}{b\overline{\sigma}\sqrt{2\pi t_1}} \exp\left\{-\frac{1}{2}\left(\frac{\ln\frac{b_0}{b} - (\overline{\mu} - \frac{1}{2}\overline{\sigma}^2)t_1}{\overline{\sigma}\sqrt{t_1}}\right)^2\right\}.$$

We next need to incorporate how the price process evolves through time. We make the traditional assumption that prices evolve according to geometric Brownian motion, that is the price at time t, denoted b_t, evolves according to the stochastic differential equation:

$$\frac{db}{b} = \overline{\mu}dt + \overline{\sigma}dz,$$

[16]For two meetings and a normalized decision cycle with the first meeting occuring at $t_0 = 0$ and the cycle ending at $t_2 = 1$, the only (timing) choice variable is t_1.

where dz represents an increment of the standard Weiner process. It is a consequence of Ito's Lemma that given an initial starting value for the input price b_0, the price realized after an elapse of time t_1, is log-normally distributed with mean equal to:

$$\ln b_0 + (\overline{\mu} - \frac{1}{2}\overline{\sigma}^2)t_1 \tag{11}$$

and standard deviation

$$\overline{\sigma}\sqrt{t_1}.$$

In order to simplify further the presentation we shall take b_0 to be a numeraire, so that $b_0 = 1$. Let us now consider how the optimal censor of section 2 is constructed in this more general setting.

3.2 The Censoring Process

Recalling (6) when b_0 takes the value 1, we see that the optimal censor $\widetilde{b}$ is chosen to satisfy:

$$1 = \int_0^{\widetilde{b}} bq(b)db + \widetilde{b}\int_{\widetilde{b}}^{\infty} q(b)db \tag{12}$$

and immediately see the right-hand side is the expected pay-off from holding long an asset and short a call on an asset with strike price $\widetilde{b}$. Given our price process assumption, we may refer to the standard Black-Scholes call option valuation to evaluate (12). The intuition is simple, the strike-price needs to be selected so that the opening price of the asset $b_0 = 1$ balances the difference between the expected price (assumed to exceed the opening price) and the option premium. Using standard finance notation adapted for our formulation let:

$$\begin{aligned} d_1 &= d_1(b,\mu,\sigma^2) = d_1(b,\overline{\mu}t_1,\overline{\sigma}^2 t_1) = \frac{\ln\left(\frac{1}{b}\right) + (\overline{\mu} + \frac{1}{2}\overline{\sigma}^2)t_1}{\overline{\sigma}\sqrt{t_1}} \qquad (13) \\ d_2 &= d_2(b,\mu,\sigma^2) = d_2(b,\overline{\mu}t_1,\overline{\sigma}^2 t_1) = d_1 - \overline{\sigma}\sqrt{t_1} = \frac{\ln\left(\frac{1}{b}\right) + (\overline{\mu} - \frac{1}{2}\overline{\sigma}^2)t_1}{\overline{\sigma}\sqrt{t_1}} \end{aligned}$$

with $N(.)$ representing the cumulative probability distribution function for a standardized normal variable then we can now rewrite (12) as requiring:

$$1 = e^{\mu}N(-d_1(\widetilde{b})) + \widetilde{b}N(-d_2(\widetilde{b})), \tag{14}$$

where $\mu = \overline{\mu}t_1$ and $\sigma^2 = \overline{\sigma}^2 t_1$; for convenience we will also write $\widetilde{d_i} = d_i(\widetilde{b})$. What is important to note at this stage is that, whereas we have just briefly exploited established results from the finance literature, the next step is distinct from that context. In particular, we note that in our analysis $\widetilde{b}$ is equivalent to the unique strike price in a Black-Scholes analysis that solves a subsidiary first-order condition; that is, we exploit the Black-Scholes approach to determine a valuation equation, but this is only a simple part of the analysis. Our contribution is to identify the strike price that satisfies the subsidiary first-order condition which an optimal censor must satisfy, and then, from this result, see what are the implications for optimal choice of the timing t_1^* for the meeting.

Before turning to consideration of the specification of the returns function, let us briefly consider two properties of the optimal hedge which aid the development of intuition. It can be shown[17] that:

$$(i) \qquad \frac{\partial u^*(\mu, \sigma)}{\partial \mu} > 0, \tag{15}$$

that is, the optimal hedge is increasing in the drift and

$$(ii) \qquad \frac{\partial u^*(\mu, \sigma)}{\partial \sigma} < 0, \tag{16}$$

that is the optimal hedge is decreasing in the standard deviation. The first statement has the simple interpretation that the optimal hedge quantity u^* increases the more the input price is expected to rise. Similarly, the second statement reflects the fact that the greater the standard deviation the greater the value of waiting, and hence the smaller the optimal (advance) hedge quantity. Having determined these properties for the optimal hedge u^*, the next task is to consider the form of the returns function.

3.3 The Returns Function

In order to simplify the presentation we introduce the notation:

$$\begin{aligned}
I_1(\widetilde{b}) &= \int_0^{\widetilde{b}} q(b)db, \\
I_{2,n}(\widetilde{b}) &= \int_{\widetilde{b}}^{\infty} b^n q(b)db, \\
J_{2,n}(\widetilde{b}) &= \int_0^{\widetilde{b}} b^n q(b)db,
\end{aligned}$$

and we see substituting (6) in (5) that, applying this new notation, we can express $F_1(u, b_0)$ as:

$$F_1(u, b_0) = J_{2,-1}(\widetilde{b}) + \frac{1}{\widetilde{b}}(1 - I_1(\widetilde{b})). \tag{17}$$

We are now finally in a position to write down[18] the total period returns, which applying (2) is given simply by:

$$R(t_1) \;=\; t_1 + (1 - t_1)F_1(u, b_0) \tag{18}$$

[17] Details available from the authors.

[18] We note that for the general Cobb-Douglas instantaneous returns function x^β the indirect returns function takes the form:

$$F_1(u, b_0) = J_{2,-\gamma}(\widetilde{b}) + \frac{1}{\widetilde{b}^\gamma}(1 - I_1(\widetilde{b})),$$

where $\gamma = \frac{\beta}{1-\beta}$.

$$= \quad t_1 + (1-t_1)[J_{2,-1}(\tilde{b}) + \frac{1}{\tilde{b}}(1 - I_1(\tilde{b}))]. \tag{19}$$

To illustrate the function, Figure 1 below presents a numerical plot of $R(t_1)$ (we note that the curve may either be concave or convex initially).

Place Figure 1 here

We now derive the first-order condition from (18) to be:

$$0 = R'(t_1) = 1 - F(t_1) + (1-t_1)F'(t_1),$$

i.e.

$$F(t_1) - 1 = (1-t_1)F'(t_1). \tag{20}$$

Applying (20) in Appendix A, we derive an implicit relationship $K(\mu, \sigma^2, t_1) = 0$ between μ and σ^2 which depends on t_1 in a simple way. In particular a routine calculation shows that:

$$K(\mu, \sigma^2, t_1) = \frac{t_1}{1-t_1} + e^{-\mu+\sigma^2}(\sigma^2 N(-\tilde{d}_2 + \sigma) + K_0) \tag{21}$$

with

$$\begin{aligned} K_0 \quad = \quad & -(\mu + \frac{t_1}{1-t_1})N(\sigma - \tilde{d}_2) + \frac{1-3t_1}{2-2t_1}N(\tilde{d}_2)e^{-\sigma\tilde{d}_2 - \frac{\sigma^2}{2}} \\ & + N(-\tilde{d}_2 - \sigma)(\mu + \frac{1}{2})e^{-2\sigma\tilde{d}_2 - 2\mu} - \frac{1}{2}e^{-2\sigma\tilde{d}_2 - \mu}), \end{aligned}$$

where the leading term in $K(.)$ is $e^{-\mu+\sigma^2}$ comes from the $E[\frac{1}{b}]$ term assuming log normality. Given (21), a critical $\overline{\mu}, \overline{\sigma}^2$ frontier may now be computed for any choice of t_1 as the solution to:

$$K(\overline{\mu}t_1, \overline{\sigma}^2 t_1, t_1) = 0.$$

In particular we can set $t_1 = 0.5$ and then are able to identify the pairs of instantaneous mean and standard deviation that satisfy:

$$K(0.5\overline{\mu}, 0.5\overline{\sigma}^2, 0.5) = 0.$$

We shall describe the zero contour of $K(0.5\overline{\mu}, 0.5\overline{\sigma}^2, 0.5)$ as the **periodic efficiency frontier** since along the frontier any variation in $\overline{\mu}$ is exactly offset by variations in $\overline{\sigma}^2$ in such a manner that $t_1 = \frac{1}{2}$ remains the optimal choice of timing the second meeting as illustrated below:

Place Figure 2 here.

Given some empirically observed pair $(\overline{\mu}, \overline{\sigma}^2)$, the chance of $\overline{\mu}$ and $\overline{\sigma}^2$ exactly offsetting the advance hedging and waiting advantages is small. That is, scheduling meetings in a balanced fashion will typically not be optimal. However, we continue to consider the periodic efficiency frontier because it aids intuition for results that will be derived shortly. In order to develop intuition let us consider first the principal qualitative feature of the frontier: its positive slope. To understand its form it is perhaps easiest to consider first some representative points located off the frontier. Suppose we are below the periodic efficiency frontier $K(.)$ with a (relatively) large input price drift and small variance. In this case, t_1 is optimally chosen to be less than $\frac{1}{2}$. To understand the intuition we see that the management accountant has to trade-off the fact that when waiting to reconvene a meeting, the expected input price rises relatively fast, while the value of waiting determined by the variance does so relatively slowly. That is, for relatively (to σ^2) large positive price drift μ, the management accountant should recommend a meeting 'soon' (i.e. before $t_1 = \frac{1}{2}$), in order to reset advance purchase agreements before the input price rises too much. Conversely, above the periodic efficiency frontier $K(.)$ the input price is expected to increase only relatively slowly, and thus relatively high price variance considerations should lead the management accountant to recommend waiting beyond $t_1 = \frac{1}{2}$ before the follow-up meeting is convened. This discussion provides us with intuition for the positive slope of the frontier. If, as the price drift increases, it is to remain optimal for the meetings still to be periodic, the variance must also rise commensurately, i.e. the increased incentive to meet earlier, caused by increasing drift, must be exactly offset by an increased value of waiting, arising from increase in the variance.

To summarize then, we have shown that for a simple two meeting model, a periodic meetings policy is unlikely to be optimal given value of waiting considerations. At issue then is how does this speak to the generally observed practice of balanced / periodic meetings policies. We next note that, in general, meetings are costly and hence, if one is concerned about optimizing over meeting frequency, these cost considerations should be included in the model specification. Intuitively we can see that introducing such costs can result in a discrete optimal number of meetings; but this in itself is unlikely to increase the likelihood of periodic meetings policy being optimal. Indeed it is not until we introduce a differential meetings cost that we can identify a wide class of settings in which periodicity would be optimal. The main thrust of the following section is to develop a bound for the differential cost that would insure that periodicity is endogenously optimal.

4 Multi-Period Analysis with $n > 1$ Review Meetings

This section is organized as follows. We begin by discussing the generalized censor which applies when an arbitrary number of meetings n is allowed. Assuming Cobb-Douglas returns technology we explain that the generalized censor is simply a multiplicative variant of the previously developed $(n = 2)$ censor[19]. In the next subsection we develop an approximation for

[19]Recall that with $n = 2$ and the first meeting (by convention) set to occur at $t_0 = 0$, we only require the determination of one censor which corresponds to the optimally chosen t_1.

the returns functions for an optimal flexible meeting policy and contrast this with the returns with a periodic policy. In the final subsection we introduce a unit periodic meeting cost a which allows us to define the optimal number of meetings n^*. Next we introduce a differential cost parameter θ which captures the assumption that the unit cost of meeting flexibly may be greater than the unit cost of meeting periodically. We identify a critical value for this differential cost, $\widehat{\theta}$, such that mangement are indifferent between meeting (n^* times) periodically, or, instead, flexibly[20]. We derive the critical value $\widehat{\theta}$ parametrically and hence are able to establish economic conditions under which management optimally choose to meet periodically. Thus, similar in spirit to Huberman (1988), though within a very different model setting, we establish bounds for incremental meeting costs under which periodicity is endogenously optimal for decision makers. However, our model also allows us to go further, because we can consider the effect of an increase in risk (measured by $\alpha = \sigma^2 - \mu$) on the desirability of flexibility. In particular, we show that starting at some value for α and θ, if at first periodicity is optimal with $\overline{n}$ meetings as we increase α, flexibility becomes optimal; then in some cases this will require a reduction in the optimal number of meetings to a number below $\overline{n}$. Thus managing a shift to higher risk is not simply a case of switching to flexibility and maintaining or increasing the total number of meetings. In certain circumstances, as defined by our optimization model, a switch to flexibility may also require a reduction in the number of meetings.

4.1 The n-Period Censor

In our simple earlier model we showed that it was beneficial to constrain (relative to the single meeting strategy) advance purchase agreements set at the opening review meeting, since there was some value from waiting. At the subsequent review meeting additional advance purchase agreements could be entered into in a flexible fashion, depending upon the prevailing input price conditions observed at that time. Given the flexibility (waiting) value that this additional meeting facilitated, it is clear that costless additional meetings would also have incremental value. However, typically meetings are not costless[21], and in practice incremental meeting frequency costs will need to be traded off against incremental flexibility value. Before considering the specification of such incremental costs we choose first to concentrate upon the nature of the incremental payoff benefits arising from additional meetings. That is, we need in a general setting to consider with n arbitrary meetings how the optimal censor should be set in the first period, in order to determine advance purchase agreements for the forthcoming periods.

To guide intuition we commence by considering the simplest extension, that of $n = 3$ with an initial meeting at t_0 and two subsequent meetings at t_1 and t_2. With two subsequent review meetings following the opening meeting the choice of timings t_1 and t_2 for the two meetings divides the decision interval into three production intervals as follows:

[20]In general the optimal number of flexible meetings need not be n^*. Our main concern here is simply that periodicity dominates. We will subsequently return to the issue of differences between the optimal number of meetings under the two regimes.

[21]See Kelly and Chapanis (1982) for a study of the scheduling of 'costly' managerial meetings.

$$\begin{array}{ccll} t_0 & \to & t_1 & \text{the first production interval,} \\ t_1 & \to & t_2 & \text{the second production interval,} \\ t_2 & \to & t_3 & \text{the third production interval,} \end{array}$$

where by assumption

$$t_0 = 0 < t_1 < t_2 < t_3 = 1.$$

It is important to note that we make no assumption about the respective lengths of the intervals, only that the total length is normalized to unity. Thus this specification is quite general and allows for the intervals to be of unbalanced length. Introducing generalized notation to incorporate the effect of the additional meeting, we denote the purchase agreed at the initial $t_0 = 0$ meeting as v_0 which is chosen so as to include:

$$\begin{array}{rl} & \text{a hedge of } u_0 \text{ carried forward from } t_0 \text{ to } t_1 \\ \text{and} & x_0 \equiv v_0 - u_0 \text{ for use in the first period starting at } t_0 = 0 \end{array} \tag{22}$$

The time order line is given as follows:

Time Order Line

At t_0 :
Observe the current input price b_0.
Predict $E(b_1)$ and $E(b_2)$ the expected prices for the following accounting intervals.
Assess initial requirements for the three periods as $v_0 = x_0 + u_0$.
Agree with the supplier to fix the price of v_0 units of input at b_0, x_0 for delivery at t_0, u_0 for delivery at t_1.
Between t_0 and t_1 use x_0 units of input in production.

At t_1 :
Observe b_1.
Predict $E(b_2)$ the expected price for the following accounting interval.
Assess new requirements for the two periods as $v_1 = x_1 + u_1$, where u_1 is a hedge for period starting t_2.
If $x_1 > u_0$, agree with the supplier to fix the price of $v_1 - u_0$ units of input at b_1, with a part of it, namely $u_0 + u_1 - x_1$, deliverable at t_2 ;
otherwise, determine how to apportion u_0 between the two periods via a first order condition.
Between t_1 and t_2 use x_1 units of input in production.

At t_2:
Observe b_2.
Assess new requirements for the final period as $v_2 = x_2$.

If $x_2 > u_1$, agree with the supplier to fix the price of $v_2 - u_1$ units of input at b_2.

Between t_2 and $t_3 = 1$:
Use x_2 units of input in production.

At t_3 the decision horizon ends.

Let us now consider the specification for the returns functions over the respective intervals, when x_0, x_1 and x_2 are the quantities of the direct inputs utilized in the three production intervals. With a constant coefficent production function $2\sqrt{x_j}$, a constant rate of production is optimal[22]. Thus, for $j = 0, 1, 2, 3$ with inputs being consumed at a constant rate in the inter-meeting interval, the return over a production interval is given by the unit period return multiplied by the length of the interval:

$$2\sqrt{\frac{x_j}{(t_{j+1} - t_j)}}.(t_{j+1} - t_j) = 2\sqrt{x_j.(t_{j+1} - t_j)},$$

that is, the previously identified returns function is amended by a new multiplicative factor $\sqrt{(t_{j+1} - t_j)}$. In the spirit of dynamic programming, before trying to solve the overall production and hedging problem commencing at t_0, suppose for the moment that we were considering the problem at t_1 with no stock in hand. Given only one intermediate meeting before the end, we can apply what we learnt about the censoring problem in the earlier case as follows. Equivalent to our development of equations (2) through (6), we have[23]:

$$\begin{aligned} u_1^* &= \arg\max 2\sqrt{x_1.(t_2 - t_1)} - b_1(x_1 + u_1) \\ &\quad + F_2(u_1, b_1) \end{aligned}$$

and the optimal hedge is given by:

$$u_1^* = \frac{(t_3 - t_2)}{\widetilde{b}_2{}^2} \tag{23}$$

were $\widetilde{b}_2(.)$ solves

$$b_1 = \int_{\underline{b}_2}^{\widetilde{b}_2(.)} b_2 dq(b_2, t_2 - t_1) + \sqrt{\frac{(t_3 - t_2)}{u_1}} \int_{\widetilde{b}_2(.)}^{\overline{b}_2} b_2 dq(b_2, t_2 - t_1). \tag{24}$$

that is, in this generalized case we have (6) amended by the $\sqrt{(t_{j+1} - t_j)}$ factor and indicated the dependence of the density function on the time lapse. Next we exploit the fact that given

[22] Alternatively in the presence of a stochastic returns parameter a_t implying a production function of the form $a_t\sqrt{x_t}$ this would not generally be true. Details are available from the authors.

[23] Note that technically we should define the stock being carried forward as, say, $\widehat{u}_1$ so as to differentiate from our earlier defined u_1 (which is chosen in the presence of an initial stock u_0). We do not, so as to simplify the notation, and because after the explanatory discussion we do not use " $\widehat{u}_1$ " again.

the input price is assumed to be lognormally distributed, the price at t_2 given we commence t_1 by observing b_1 obeys the law of proportionate effect, that is:

$$b_2(.) = b_1.g_{2,(t_2-t_1)}$$

and hence we can always express a censor as the product of the opening input price times a proportionality factor. That is we can re-express (23) as:

$$u_1^* = \frac{(t_3 - t_2)}{(b_1.\widetilde{g}_{2,(t_2-t_1)})^2},$$

where $b_1.\widetilde{g}_{2,(t_2-t_1)}$ solves (24). The desirability of this transformation becomes clear when we proceed back to the next earlier period. Exploiting the fact that we chose

$$b_0 = 1,$$

we can, by repeated application of the law of proportionate effect, express

$$b_1(.) = g_{1,t_1-t_0}, \quad b_2(.) = g_{1,t_1-t_0}.g_{2,t_2-t_1}.$$

This homogeneity can then be traced inductively through $F_1(u_0, b_0)$ to give[24]:

$$u_1^* = \frac{(t_3 - t_2)}{(\widetilde{g}_{1,t_1-t_0}.\widetilde{g}_{2,t_2-t_1})^2},$$

that is, at $t_0 = 0$ we have:

$$\begin{aligned} v_0 - u_0^* &= \frac{(t_1 - t_0)}{b_0^2} = (t_1 - t_0), \\ u_0^* &= \frac{(t_3 - t_2)}{(\widetilde{g}_{1,t_1-t_0}.\widetilde{g}_{2,t_2-t_1})^2} + \frac{(t_2 - t_1)}{(\widetilde{g}_{1,t_1-t_0})^2}. \end{aligned}$$

Generalizing to the case of n meetings, it can then be shown that the optimal hedge in the first period satisfies:

$$u_0^* = \sum_{j=1}^{n} \left(\frac{(t_{j+1} - t_j)}{\prod_{k=1}^{j} (\widetilde{g}_{k,t_k-t_{k-1}})^2} \right). \tag{25}$$

Having determined the analytical form of the optimal first period hedge when there are n meetings, we can now turn to consider the returns functions in these generalized settings.

[24] See Gietzmann and Ostaszewski (1999)b. We present these results here for completeness and subsequently show that a simpler approximating approach can be used.

4.2 The Returns Function With n-Meetings

In general, an analytical statement concerning the exact form of the generalized optimal returns function is not possible, given the complex nature of interactions beween the various periods advance hedging decisions. However, in one reasonable setting[25] we can make good approximations to the analytical form. In particular, if we assume that $\overline{\sigma}^2$ is large relative to $\overline{\mu}$ then, as we can see applying (16), the optimal hedge for n periods ahead set at t_0 will approach zero. Under this assumption we can then approximate the maximum expected return $\widetilde{R}_n(.)$. Within this setting we proceed to find how the optimal timing for meetings should be scheduled. Our first observation is that the the inter-meeting periods are naturally unbalanced with longer intervals at the beginning becoming progressively shorter (see Appendix C).

We begin by considering the expected value arising from an arbitrary timing choice $0 < t_1 < ... < t_{n-1} < 1$ given by the function:

$$R_n(t_1, ..., t_{n-1}) = \sum_{i=0}^{n-1} (t_{i+1} - t_i) E_i \left(\frac{1}{b_i} \right)$$

where E_i is the expectations operator applied to the reciprocated price[26] at time t_i:

$$E_i \left(\frac{1}{b_i} \right) = \int_0^\infty \frac{q_i(b) db}{b},$$

with $q_i(b)$ denoting the time t_i distribution of price. That is, a policy is pursued under which no hedge is carried forward, and in each period, production is conditioned upon the observed current input price. Given our log-normality assumption[27] and letting:

$$\alpha = \overline{\sigma}^2 - \overline{\mu}$$

[25]We will justify this assumption below. To see how our results derived below relate to the case when a hedge is carried forward (under small σ^2 relative to μ) we note that the effect of a hedge is to lengthen the first period before a review and hence the underlying dynamic is a **telescoping effect**. Thus, when considering the effect of moving from an unhedged situation to one that allows hedging, one simply needs to assess the relative magnitude of the telescoping effect.

[26]Recall that the return $\pi = 2\sqrt{u} - bu$ for $u = \frac{1}{b^2}$ becomes $\pi = \frac{1}{b}$, that is to maximize expected return π we need to maximize over the expected inverse of the input price.

[27]Under log-normality

$$\log b \sim N(\overline{\mu} - \frac{1}{2}\overline{\sigma}^2)t, \overline{\sigma}^2 t)$$

so

$$E(\frac{1}{b}) = \int_0^\infty \frac{1}{b} q(b) db = \int_{-\infty}^\infty e^{-(\sigma w + m)} \frac{e^{-\frac{w^2}{2}}}{\sqrt{2\pi}} dw$$

with

$$w = \frac{\ln b - m}{\sigma}, \quad b = e^{\sigma w + m}, \quad \sigma = \overline{\sigma}\sqrt{\tau}, \quad m = (\overline{\mu} - \frac{1}{2}\overline{\sigma}^2)\tau$$

completing the square and rearranging then gives

$$E(\frac{1}{b}) = e^{\sigma^2 - \mu} = e^{\alpha}.$$

we have[28] that:

$$R_n(.) = \sum_{i=0}^{n-1}(t_{i+1} - t_i)e^{\alpha t_i}. \tag{26}$$

We assume $\alpha \geq 0$ as we are interested in high risk effects. Under this assumption there exists a maximum value for R_n. To derive the optimal payoff $\widehat{R}_n$ in this setting we need to insure that the individual t_i 's are chosen optimally. The $n-1$ first-order conditions[29] for the optimal choice of timing t_i^* for each meeting are given by:

$$e^{\alpha t_{i-1}^*} - e^{\alpha t_i^*} + \alpha(t_{i+1}^* - t_i^*)e^{\alpha t_i^*} = 0. \tag{27}$$

Summing over the timing indicies $i = 1, ..., n-1$ we obtain:

$$1 - e^{\alpha t_{n-1}^*} + \alpha(\widehat{R}_n(.) - t_1^*) = 0,$$

or:

$$\widehat{R}_n(t_1^*, .., t_{n-1}^* | \alpha) = t_1^* + \frac{e^{\alpha t_{n-1}^*} - 1}{\alpha}. \tag{28}$$

What is particularly interesting about this statement concerning the optimal payoff is that it is expressable in terms of the optimal length of the first and last periods as well as the parametric value of α.

Let us now turn to consider the payoff that could be achieved if meetings were chosen to occur periodically, that is all inter-meeting periods were set such that:

$$t_i = \frac{i}{n} \qquad \text{for} \qquad i = 1, ..., n.$$

In this special case (26) becomes:

$$R_n^{=}(\alpha) = \frac{1}{n} + \frac{1}{n}e^{\alpha/n} + ... + \frac{1}{n}e^{(n-1)\alpha/n}.$$

Summing terms as a geometric progression then gives:

$$R_n^{=}(\alpha) = \frac{e^{\alpha} - 1}{n(e^{\frac{\alpha}{n}} - 1)} \tag{29}$$

and hence we have a simple form for computing the payoff with n periodic meetings. Clearly:

$$\widehat{R}_{n+1}(t_1^*, .., t_n^* | \alpha) > \widehat{R}_n(t_1^*, .., t_{n-1}^* | \alpha) \geq R_n^{=}(\alpha) \tag{30}$$

We have (30) holding for any arbitrary n and, in particular, the left-hand side inequality establishes that when meetings have no cost, additional meetings always give rise to a net

[28] Recall the leading term of (21).

[29] In order to simplify the presentation we do not show explicitly the formal Lagrangian analysis which incorporates the timing constraint that $t_i < t_{i+1}$ is in fact satisfied.

benefit; the right-hand side establishes that, again, if meetings are costless a flexible meetings policy will typically dominate a periodic policy. In the following subsection we shall relax this unrealistic assumption concerning meeting costs and consider how the analysis can be conducted when it is recognized that meetings are costly to schedule. The effect of this will be that we can then identify an optimal number of meetings, n^*, above which the incremental cost of meeting outweighs the incremental benefit.

4.3 The Optimal Number of Meetings n^* and the Dominance of Periodicity

Let us assume that the unit cost of scheduling a periodic meeting is simply some fixed value a. In order to develop a functional form for the cost of non-periodic meetings, assume that choosing to lengthen or shorten the interval between meetings (introducing flexibility) entails an additional cost θa times the absolute difference from an overall median interval of length Δ; that is, we treat the median interval as comparable to a standard interval of the periodic setting, on the grounds that $\lim_{n\to\infty} \frac{\Delta}{n}$ exists. The extra scheduling cost of meeting flexibly is thus given by $W(.)$, defined as:

$$\begin{aligned} W(.) &= \sum_{i<n/2} a\theta(t_{i+1} - t_i - \Delta) + \sum_{i\geq n/2} a\theta(\Delta - (t_{i+1} - t_i)) \\ &= a\theta\{\sum_{i<n/2} (t_{i+1} - t_i) - \sum_{i\geq n/2} (t_{i+1} - t_i)\}. \end{aligned}$$

Recalling that the length of the meeting intervals are monotonically decreasing (see Appendix C), if the length of the first interval between meetings is given by:

$$t_1 - t_0 = \frac{\lambda(\alpha, n)}{n},$$

then adding and subtracting $\sum_{i<n/2}(t_{i+1} - t_i)$ from the bracketed term gives:

$$\begin{aligned} W(a, \theta, \lambda(a, n)) &= a\theta\{2 \sum_{i<n/2} (t_{i+1} - t_i) - 1\} \\ &\leq a\theta\{2 \frac{n}{2} \frac{\lambda(\alpha, n)}{n} - 1\} \\ &= a\theta(\lambda(\alpha, n) - 1) \\ &\leq a\theta(\lambda(\alpha) - 1), \end{aligned}$$

where $\lambda(\alpha)$ denotes the following (monotone) limit:

$$\lambda(\alpha) = \lim_{n\to\infty} \lambda(\alpha, n), \tag{31}$$

which exists and has value[30] for $\alpha > 0$:

$$\lambda(\alpha) = \frac{2(e^{\alpha/2} - 1)}{\alpha}. \tag{32}$$

We will thus use the following expression as a model of the incremental costs of meeting flexibly:

$$a\theta(\lambda(\alpha) - 1) \tag{33}$$

That is, the incremental cost for n flexible meetings is θ times a measure of how unbalanced the meeting intervals are chosen. With this additional specification we can now define the optimal number of flexible meetings $\widehat{n}$ by:

$$\widehat{n} \equiv \arg\max \hat{R}_n(\alpha) - \{an + a\theta(\lambda(\alpha) - 1)\}. \tag{34}$$

This maximization is independent of θ (since $\lambda(\alpha)$ has replaced $\lambda(\alpha, n)$). The value $\widehat{n}$ can be determined[31] analytically if we approximate $\hat{R}_n(\alpha)$ by $R(n, \alpha)$ and refer to the first-order condition (see Appendix B):

$$\frac{\partial R(n, \alpha)}{\partial n} = a,$$

yielding

$$\widehat{n} \simeq n^*(\alpha, a) = \sqrt{\frac{e^{\alpha} - 1}{2a}}. \tag{35}$$

Evidently, there will be occasions when equality obtains, as $\widehat{n}$ varies over the integers.

Let us now consider the critical value for the cost factor $\widehat{\theta}$ at which periodicity becomes optimal. This is defined by

$$R^{=}_{\widehat{n}}(\alpha) - a\widehat{n} = \hat{R}_{\widehat{n}}(\alpha) - \{a\widehat{n} + a\hat{\theta}(\lambda(\alpha) - 1)\},$$

whence

$$\widehat{\theta} = \frac{\hat{R}_{\widehat{n}}(\alpha) - R^{=}_{\widehat{n}}(\alpha)}{a(\lambda(\alpha) - 1)}$$

Thus for values of $\theta > \widehat{\theta}$ periodicity is optimal. Our overall goal is to determine the critical value of θ parametrically, so since $\hat{R}_{\widehat{n}}(\alpha)$ has no simple representation we approximate[32] this term by $R^{=}_{\widehat{n}+1}(\alpha)$. In Appendix C we show that provided α is not too large we have for any n that:

$$R^{=}_{n+1}(\alpha) > \hat{R}_n(\alpha). \tag{36}$$

[30]Proof available from the authors.

[31]See Appendix B.

[32]As will be clear from the subsequent discussion of Figure 3, sometimes the comparison will need to be between an n^* flexible and a $n^* + 1$ balanced policy using $R^{=}_{n^*+2}$ as the approximation for $\widehat{R}_{n^*}$.

We may thus replace $\hat{R}_{\widehat{n}}(\alpha)$ by $R^{\equiv}_{\widehat{n}+1}(\alpha)$ and so determine a revised critical value as follows:

$$\widetilde{\theta} = \frac{R^{\equiv}_{\widehat{n}+1}(\alpha) - R^{\equiv}_{\widehat{n}}(\alpha)}{a(\lambda(\alpha) - 1)}, \tag{37}$$

where significantly, by (36), we have:

$$\widetilde{\theta} > \widehat{\theta},$$

that is when θ takes the critical value $\widetilde{\theta}$ periodicity is optimal.

To understand the dependence of the indifference-cost $\widetilde{\theta}$ on risk α in terms of economic observables we need to invoke a Taylor expansion. Recalling (29), we see from (37) that the critical θ is given by:

$$\widetilde{\theta} = \frac{\frac{e^{\alpha}-1}{(n+1)(e^{\frac{\alpha}{(n+1)}}-1)} - \frac{e^{\alpha}-1}{n(e^{\frac{\alpha}{n}}-1)}}{a(\lambda(\alpha) - 1)},$$

where $n = \widehat{n}$. Using the first two terms of a series expansion for some of the exponential terms then gives:

$$\widetilde{\theta} \simeq \frac{e^{\alpha} - 1}{2a\widehat{n}(\widehat{n} + 1)} \cdot \frac{1}{(\lambda(\alpha) - 1)}.$$

This formula exhibits two trends. First, it is readily verified that $\widetilde{\theta}$ with a held constant is increasing with α over any interval in which $\widehat{n}$ is constant. Second, when we limit our attention to those values of α where $\widehat{n} = n^*$ we may apply (35) and (31) to give the approximation

$$\widetilde{\theta} \simeq \frac{e^{\alpha} - 1}{(\alpha + \sqrt{2a\alpha})} \cdot \frac{1}{(\lambda(\alpha) - 1)}.$$

and some routine calculus establishes that the derivative with respect to α of the right-hand side is initially negative and thereafter becomes positive.

This occurs because the defining equation for $\widetilde{\theta}$ (when a is fixed) shows it to be a ratio of benefit from one more periodic meeting to a cost of flexibility measured by $(\lambda(\alpha) - 1)$; we see from the above analysis that for large α the benefits behave like e^{α}/α whereas the said costs behave like $e^{\alpha/2}/\alpha$ (refer to (32)). Overall the benefits outstrip the costs exponentially. For small α the formulas show the benefits to be indifferent to α, whilst the costs behave like α itself. This means that, as risk α rises, the cost of indifference $\widetilde{\theta}$ first falls and then rises. Although $\widehat{n}$ does not vary continuously, the overall trend described above certainly holds at values α where n^* agrees with the integer $\widehat{n}$.

At a first pass, one may be tempted to suggest that such observations suggest the following, simple intuitive policy rule. Keeping the incremental cost $\widetilde{\theta}$ constant, as risk measured by α increases there is a shift from periodic to flexible meetings and an increase in the frequency n of meetings. That is, if this were true the optimal response to increasing risk would be to meet more frequently in a flexible fashion.

However, it is relatively straightforward to present a counter-example which shows the above seemingly intuitive claim is not robust. Applying (28) and (29), Figure 3 below plots the optimal meetings regime as a function of θ and α.

Place Figure 3 here

Starting at point A, holding θ constant and increasing α we see that the optimal response to increased risk is to switch from meeting ten times periodically to a flexible basis with nine meetings. That is, for some parameter regimes the optimal response is to switch to a flexible less frequent meetings policy. The intuition for the result is as follows. As the incremental cost

θ of meeting flexibly rises we know that the optimal number of periodic meetings is unaffected while the number of flexible meetings reduces. Thus at a sufficiently high θ, holding α constant, the optimal number of balanced meetings will be greater than the optimal number of flexible meetings. Assuming that a point like A is defined by an initial α value that insures that periodicity dominates flexibility, as α is increased a switch will occur when flexibility will come to dominate and the optimal number of meetings may be commensurately reduced.

This counter-example shows one of the major advantages of working within an optimization framework. Our model shows how subtle the optimal response strategy is to increasing risk. Our modelling shows that the application of simple intuitive heuristics may lead to suboptimization with over-incurrence of meeting costs. Thus, our model contributes to development of an understanding of why some organizations do not simply implement relatively straightforward revisions / adaptations in organizational procedures in response to increasing risk.

5 CONCLUSION

One of the most powerful decision aids that management accountants can provide management with is timely information at key meetings. However, given opportunity costs associated with convening meetings of cross functional management teams, choice of the optimal timing of provision of information is a complex task. In this respect there exists a host of different costs associated with convening managerial meetings and simultaneously opportunity costs of delay (e.g. misallocation of resources in stochastically evolving environments). We have modelled this trade-off by introducing a simple meeting cost function and combining value of waiting considerations with hedging incentives in an inflationary price environment.

Our analysis shows how to determine endogenously the optimal length of management accounting periods. Moreover, the results motivate us to suggest that management accountants need to develop a thorough appreciation of two central issues. First, our results show that a rich set of period lengths is needed to attain optimality. Second, we have stressed how sensitive the exact form of these results is to assumptions concerning the relative costs of convening meetings. Simply observing that 'many' organizations used balanced / periodic accounting periods is not sufficient evidence, in our view, to recommend unquestioned adoption. Before deciding upon timing of accounting periods, we suggest, management accountants need consider critically, and monitor, the underlying economic environment in relation to the required changes in meetings policy. In a more directly constructive sense, we have established that the optimality of a periodic meetings policy depends critically upon the incremental cost of switching from a periodic to a flexible scheduling routine. In our analysis the critical incremental cost has been derived parametrically so that, in principle, it can be related to empirically observed parameter values. Moreover, we have established that, faced with an increase in risk, while it may be optimal to shift from a periodic to a flexible meetings policy, the optimal frequency of meetings need not be non-decreasing. That is, increased risk may induce an optimal shift in meetings policy which involves a reduction in the overall frequency of meetings. This result illustrates the benefit of working within a formal optimization framework, and why management accounting theory should not be based upon models which endogenously impose periodic / balanced accounting cycles.

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Appendix A: (Calculation of four integrals)

In order to solve (20) we must compute $F'(t_1)$. Now by Leibniz's Rule (taking into account cancellations) we have

$$\begin{aligned}
&\frac{\partial}{\partial t_1}\left\{\int_0^{\widetilde{b}_{t_1}} \frac{q(b,t_1)}{b}db + \int_{\widetilde{b}_{t_1}}^{\infty} \frac{q(b,t_1)}{\widetilde{b}(t_1)}db\right\} \\
= &\int_0^{\widetilde{b}_{t_1}} \frac{1}{b}\frac{\partial q}{\partial t_1}db + \int_{\widetilde{b}_{t_1}}^{\infty} \frac{1}{\widetilde{b}(t_1)}\frac{\partial q}{\partial t_1}db - \frac{1}{\widetilde{b}(t_1)^2}\frac{d\widetilde{b}(t_1)}{dt_1}\int_{\widetilde{b}_{t_1}}^{\infty} q(b,t_1)db.
\end{aligned}$$

Likewise differentiating the censor equation with respect to the interval timing choice variable gives:

$$0 = \int_0^{\widetilde{b}_{t_1}} b\frac{\partial q}{\partial t_1}db + \int_{\widetilde{b}_{t_1}}^{\infty} \widetilde{b}(t_1)\frac{\partial q}{\partial t_1}db + \frac{d\widetilde{b}(t_1)}{dt_1}\int_{\widetilde{b}_{t_1}}^{\infty} q(b,t_1)db \tag{38}$$

and eliminating between the two we get:

$$F'(t_1) = \int_0^{\infty} \frac{1}{\left(b\wedge \widetilde{b}(t_1)\right)}\frac{\partial q}{\partial t_1}db + \frac{1}{\widetilde{b}(t_1)^2}\int_0^{\infty}\left(b\wedge \widetilde{b}(t_1)\right)\frac{\partial q}{\partial t_1}db. \tag{39}$$

A routine calculation establishes that:

$$2t_1\frac{\partial q}{\partial t_1} = -q + \left(\frac{(\ln b)^2 - (\mu - \frac{1}{2}\sigma^2)^2}{\sigma^2}\right)q.$$

Thus introducing $K_1(.)$ and $K_2(.)$ notation below, we are interested in computing:

$$\begin{aligned}
\int_0^{\infty}[b\wedge \widetilde{b}(t_1)]\frac{\partial q}{\partial t_1}db &= \int_0^{\infty}[b\wedge \widetilde{b}(t_1)]\left[-q + q\left(\frac{(\ln b)^2 - (\mu - \frac{1}{2}\sigma^2)^2}{\sigma^2}\right)\right]db \\
&= -\int_0^{\infty}[b\wedge \widetilde{b}(t_1)]qdb \\
&\quad + \int_0^{\infty}[b\wedge \widetilde{b}(t_1)]\left(\frac{(\ln b)^2 - (\mu - \frac{1}{2}\sigma^2)^2}{\sigma^2}\right)qdb \\
&= -1 + \int_0^{\widetilde{b}(t_1)} b\left(\frac{(\ln b)^2 - (\mu - \frac{1}{2}\sigma^2)^2}{\sigma^2}\right)qdb \\
&\quad + \int_{\widetilde{b}(t_1)}^{\infty} \widetilde{b}(t_1)\left(\frac{(\ln b)^2 - (\mu - \frac{1}{2}\sigma^2)^2}{\sigma^2}\right)qdb \\
&= -1 + (K_1(\widetilde{b}(t_1),\mu,\sigma^2) + K_2(\widetilde{b}(t_1),\mu,\sigma^2))
\end{aligned}$$

and also:

$$\begin{aligned}
\int_0^{\infty}\frac{1}{[b\wedge \widetilde{b}(t_1)]}\frac{\partial q}{\partial t_1}db &= \int_0^{\infty}\frac{1}{[b\wedge \widetilde{b}(t_1)]}\left[-q + q\left(\frac{(\ln b)^2 - (\mu - \frac{1}{2}\sigma^2)^2}{\sigma^2}\right)\right]db \\
&= -F(t_1) + \int_0^{\infty}\frac{1}{[b\wedge \widetilde{b}(t_1)]}\left[q\left(\frac{(\ln b)^2 - (\mu - \frac{1}{2}\sigma^2)^2}{\sigma^2}\right)\right]db \\
&= -F(t_1) + K_3(\widetilde{b}(t_1),\mu,\sigma^2) + K_4(\widetilde{b}(t_1),\mu,\sigma^2)
\end{aligned}$$

where the two new variables are analogous to our earlier ones. Thus returning to (20) by (39) we have:

$$\begin{aligned}
\frac{F(t_1)-1}{1-t_1} &= F'(t_1)\\
&= \int_0^\infty \frac{1}{[b\wedge \widetilde{b}(t_1)]}\frac{\partial q}{\partial t_1}db + \frac{1}{[\widetilde{b}(t_1)]^2}\int_{\widetilde{b}_{t_1}}^\infty [b\wedge \widetilde{b}(t_1)]\frac{\partial q}{\partial t_1}db\\
&= \frac{1}{2t_1}\left(-F(t_1)+K_3+K_4+\frac{1}{[\widetilde{b}(t_1)]^2}(-1+K_1+K_2)\right)
\end{aligned}$$

or

$$0=\frac{t_1}{1-t_1}-\frac{1}{2}\left(\frac{1+t_1}{1-t_1}\right)F(t_1)+\frac{K_3}{2}+\frac{-1+K_1+2K_2}{2[\widetilde{b}(t_1)]^2},$$

On substitution we get an implicit relationship $K(\mu,\sigma^2,t_1)=0$ between μ and σ^2 which depends on t_1 in a simple way. A routine calculations shows that:

$$K(\mu,\sigma^2,t_1)=\frac{t_1}{1-t_1}+e^{-\mu+\sigma^2}(\sigma^2 N(-\widetilde{d}_2+\sigma)+K_0),$$

where

$$\begin{aligned}
K_0 &= -(\mu+\frac{t_1}{1-t_1})N(\sigma-\widetilde{d}_2)+\frac{1-3t_1}{2-2t_1}N(\widetilde{d}_2)e^{-\sigma\widetilde{d}_2-\frac{\sigma^2}{2}}\\
&\quad +N(-\widetilde{d}_2-\sigma)(\mu+\frac{1}{2})e^{-2\sigma\widetilde{d}_2-2\mu}-\frac{1}{2}e^{-2\sigma\widetilde{d}_2-\mu}).
\end{aligned}$$

We commence with the simpler K_2 expression; for notational ease we surpress the explicit dependence of $\widetilde{b}$ on t_1 and we use the substitution

$$w=\frac{\ln b-(\mu-\frac{1}{2}\sigma^2)}{\sigma}.$$

Temporarily writing $w=W$ when $b=\widetilde{b}$ (so that $W=-\widetilde{d}_2$) we have

$$\begin{aligned}
K_2 &= \widetilde{b}\int_{\widetilde{b}}^\infty \left(\frac{\ln b-(\mu-\frac{1}{2}\sigma^2)}{\sigma}\right)\left(\frac{\ln b+(\mu-\frac{1}{2}\sigma^2)}{\sigma}\right)q\,db\\
&= \widetilde{b}\int_W^\infty w\left(w+\frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma}\right)e^{-\frac{1}{2}w^2}\frac{dw}{\sqrt{2\pi}}.
\end{aligned}$$

But

$$\begin{aligned}
\int_W^\infty x^2e^{-\frac{1}{2}x^2}dx &= [-xe^{-\frac{1}{2}x^2}]_W^\infty+\int_W^\infty e^{-\frac{1}{2}x^2}dx\\
&= We^{-\frac{1}{2}W^2}+\sqrt{2\pi}(1-N(W))
\end{aligned}$$

and

$$\int_W^\infty xe^{-\frac{1}{2}x^2}dx = [-e^{-\frac{1}{2}x^2}]_W^\infty = e^{-\frac{1}{2}W^2},$$

so

$$K_2 = \widetilde{b}\left\{e^{-\frac{1}{2}W^2}\left(W + \frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma}\right)\frac{1}{\sqrt{2\pi}} + (1-N(\widetilde{b}))\right\} \tag{40}$$

Similarly

$$K_1 = \int_{-\infty}^W e^{\sigma w + (\mu - \frac{1}{2}\sigma^2)} w\left(w + \frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma}\right)e^{-\frac{1}{2}w^2}\frac{dw}{\sqrt{2\pi}}$$

and defining $y = w - \sigma$ gives:

$$I_1(t_1) = e^{\mu+\frac{1}{2}\sigma^2}\int_{-\infty}^{W-\sigma}(y+\sigma)\left(y+\sigma+\frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma}\right)e^{-\frac{1}{2}y^2}\frac{dy}{\sqrt{2\pi}}.$$

Now

$$\begin{aligned}\int_{-\infty}^{W-\sigma} y^2e^{-\frac{1}{2}y^2}\frac{dy}{\sqrt{2\pi}} &= [-ye^{-\frac{1}{2}y^2}]_{-\infty}^{W-\sigma} + \int_{-\infty}^{W-\sigma}e^{-\frac{1}{2}y^2}dy \\ &= -(W-\sigma)e^{-\frac{1}{2}(W-\sigma)^2} + \sqrt{2\pi}N(W-\sigma),\end{aligned}$$

also

$$\int_{-\infty}^{W-\sigma} ye^{-\frac{1}{2}y^2}dy = [-e^{-\frac{1}{2}y^2}]_{-\infty}^{W-\sigma} = -e^{-\frac{1}{2}(W-\sigma)^2}.$$

So

$$\begin{aligned}K_1 &= e^\mu\left\{\begin{array}{c} N(W-\sigma) - \frac{1}{\sqrt{2\pi}}\left[(W-\sigma)+2\sigma+\frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma}\right]e^{-\frac{1}{2}(W-\sigma)^2} \\ +\sigma^2 + \frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma}N(W-\sigma)\end{array}\right\} \\ &= e^\mu\left\{(1+\sigma^2+\frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma})N(W-\sigma) - \frac{e^{-\frac{1}{2}(W-\sigma)^2}}{\sqrt{2\pi}}\left[(W-\sigma)+2\sigma+\frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma}\right]\right\}\end{aligned}$$

so

$$\begin{aligned}K_1 &= e^\mu(1+2\mu)N(W-\sigma) - \\ &\quad e^{(\mu-\frac{1}{2}\sigma^2)}\frac{e^{-\frac{1}{2}W^2+W\sigma}}{\sqrt{2\pi}}\left[W+\frac{2\mu}{\sigma}\right]\end{aligned} \tag{41}$$

Next, using the same substitutions as before, we have

$$K_3 = \int_0^{\widetilde{b}_{t_1}}\frac{1}{b}\left[q\left(\frac{(\ln b)^2-(\mu-\frac{1}{2}\sigma^2)^2}{\sigma^2}\right)\right]db$$

$$
\begin{aligned}
&= \int_{-\infty}^{W} e^{-\sigma w-(\mu-\frac{1}{2}\sigma^2)} w \left(w + \frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma} \right) e^{-\frac{1}{2}w^2} \frac{dw}{\sqrt{2\pi}} \\
&= e^{-\mu+\sigma^2} \int_{-\infty}^{W} w \left(w + \frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma} \right) e^{-\frac{1}{2}(w+\sigma)^2} \frac{dw}{\sqrt{2\pi}} \\
&= e^{-\mu+\sigma^2} \int_{-\infty}^{W-\sigma} (y-\sigma) \left(y-\sigma + \frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma} \right) e^{-\frac{1}{2}y^2} \frac{dy}{\sqrt{2\pi}} \\
&= e^{-(\mu-\sigma^2)} [\left\{ 1+\sigma^2 - 2(\mu - \frac{1}{2}\sigma^2) \right\} N(W+\sigma) - \frac{e^{-\frac{1}{2}(W+\sigma)^2}}{\sqrt{2\pi}} [(W+\sigma) \\
&\quad + \left(-2\sigma + \frac{2(\mu-\frac{1}{2}\sigma^2)}{\sigma} \right)]] \\
&= e^{-(\mu-\sigma^2)} [\{1-2\mu\} N(W+\sigma) - \frac{e^{-\frac{1}{2}(W+\sigma)^2}}{\sqrt{2\pi}} (W - 2\sigma + \frac{2\mu}{\sigma})].
\end{aligned}
$$

Thus

$$
\begin{aligned}
K_4(\tilde{b}(t_1), \mu t_1, \sigma^2 t_1) &= \frac{1}{\tilde{b}_{t_1}} \int_{\tilde{b}_{t_1}}^{\infty} \left[q \left(\frac{(\ln b)^2 - (\mu - \frac{1}{2}\sigma^2)^2}{\sigma^2} \right) \right] db \\
&= \frac{1}{[\tilde{b}(t_1)]^2} K_2(\tilde{b}(t_1), \mu, \sigma^2).
\end{aligned}
$$

Thus putting $\mu = \overline{\mu} t_1$, $\sigma = \overline{\sigma} t_1$, the critical $\overline{\mu}, \overline{\sigma}^2$ frontier may now be computed for any choice of t_1 as the solution to:

$$K(\overline{\mu} t_1, \overline{\sigma}^2 t_1, t_1) = 0.$$

Appendix B

We have

$$R(n,\alpha) = \frac{(e^{\alpha}-1)}{n(e^{a/n}-1)},$$

so differentiating with respect to n and writing $x = \alpha/n$, we have

$$\begin{aligned} R'(n,\alpha) &= -(e^{\alpha}-1)\cdot\frac{(e^{\alpha/n}-1)+ne^{\alpha/n}(\frac{-\alpha}{n^2})}{n^2(e^{a/n}-1)^2} \\ &= -\frac{(e^{\alpha}-1)}{\alpha^2}K(x), \end{aligned}$$

where

$$K(x) = x^2\frac{(e^x-1)-xe^x}{(e^x-1)^2} \simeq \frac{x^2}{2},$$

since the approximation $e^x = 1 + x + x^2/2$ yields

$$\frac{(e^x-1)-xe^x}{(e^x-1)^2} = \frac{x^2/2}{x^2} = \frac{1}{2}.$$

Thus the first-order condition $R' = a$, i.e.

$$K(x) = a\frac{\alpha^2}{e^{\alpha}-1},$$

approximates to

$$\frac{2a\alpha^2}{e^{\alpha}-1} = x^2 = \frac{\alpha^2}{n^2},$$

i.e.

$$\bar{n} = \sqrt{\frac{e^{\alpha}-1}{2a}}.$$

Appendix C

We begin by noting that $e^{-x} > 1 - x$ and so, rewriting the first-order condition (27), we obtain

$$\alpha(t_{i+1} - t_i) = 1 - e^{-\alpha(t_i - t_{i-1})} < \alpha(t_i - t_{i-1}),$$

i.e. the intervals are decreasing and so

$$\frac{\lambda(1,a)}{n} > \frac{\lambda(2,a)}{n} > ... > \frac{\lambda(n,a)}{n}.$$

Observe that

$$\sum_{i=0}^{n-1} (t_{i+1}^2 - t_i^2) = t_n^2 - t_1^2 = 1$$

and that

$$t_{i+1} + t_i = 2t_i + (t_{i+1} - t_i).$$

Thus we have

$$\begin{aligned}
\sum_i t_i(t_{i+1} - t_i) &= \sum_i \frac{1}{2}[(t_{i+1} + t_i) - (t_{i+1} - t_i)](t_{i+1} - t_i) \\
&= \frac{1}{2}[1 - \sum_i (t_{i+1} - t_i)^2] \\
&< \frac{1}{2}(1 - \frac{1}{n}),
\end{aligned}$$

by Jensen's inequality applied to the strictly convex function t^2. [Consider $\sum_i \frac{1}{n}(t_{i+1} - t_i)^2$ and note that $\sum_i (t_{i+1} - t_i) = 1$.] But

$$\frac{\partial \widehat{R}_n}{\partial \alpha}_{|\alpha=0+} = \sum_i t_i(t_{i+1} - t_i)e^{\alpha t_i} \,|_{\alpha=0} = \sum_i t_i(t_{i+1} - t_i)$$

and

$$\begin{aligned}
\frac{\partial R_{n+1}^{=}}{\partial \alpha}_{|\alpha=0+} &= \frac{1}{n}\{\frac{1}{n} + \frac{2}{n} + ... - \frac{n-1}{n}\} \\
&= \frac{1}{n^2}\frac{1}{2}n(n-1) = \frac{1}{2}(1 - \frac{1}{n}).
\end{aligned}$$

Thus

$$\frac{\partial[\widehat{R}_n - R_{n+1}^{=}]}{\partial \alpha}_{|\alpha=0+} < 0$$

and so

$$R_{n+1}^{=}(\alpha) > \widehat{R}_n(\alpha)$$

in an interval to the right of $\alpha = 0$.