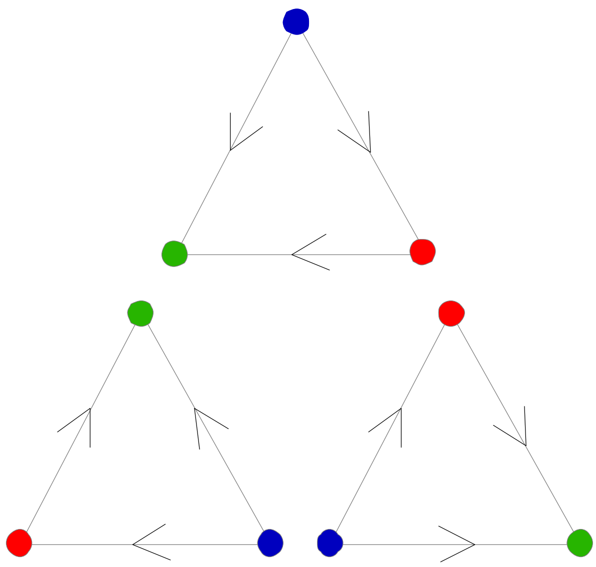
Queen Mary, University of London

The London School of Economics and Political Science

One Day Colloquia in Combinatorics



20th & 21st May 2009

Queen Mary
University of London

WEDNESDAY 20th MAY 2009

The first of the two linked colloquia in combinatorics will be held at Queen Mary, University of London on Wednesday 20th May, starting at 10.40. Everyone interested is welcome to attend any part of the event. All the talks will be held in the Maths Lecture Theatre (MLT), which can be found in the School of Mathematical Sciences.

10.00 - 10.30	*Coffee available for early arrivals in Room 102*
10.40 - 11.20	**C. Elsholtz** (Royal Holloway) *Problems in multidimensional additive combinatorics*
11.25 - 12.05	**A. Scott** (Oxford) *Triangles in random graphs*
12.05 - 13.30	***Lunch*** *(own arrangements - options on campus and nearby)*
13.30 - 14.10	**D. Ellis** (Cambridge) *Combinatorial problems on the symmetric group: some applications of non-Abelian Fourier Analysis*
14.15 - 14.55	**R. Morris** (Cambridge) *Some recent developments in bootstrap percolation*
14.55 - 15.35	***Coffee break*** *– Room 102*
15.35 - 16.15	**L. Kelly** (Birmingham) *Arbitrary orientations of cycles in oriented graphs*
16.20 - 17.00	**M. Walters** (Queen Mary) *Hamilton cycles in random geometric graphs*

Support for this event by the London Mathematical Society and the British Combinatorial Committee is gratefully acknowledged.

Multidimensional problems in additive combinatorics

Christian Elsholtz

We discuss bounds on extremal sets for problems like those below:

1. What is the largest subset of $(\mathbb{Z}/n\mathbb{Z})^r$ that does not contain an arithmetic progression of length k?
2. What is the largest subset/(multiset) of $(\mathbb{Z}/n\mathbb{Z})^r$ that does not contain n elements that sum to 0?
3. What is the largest subset of $[1, \ldots, n]^r$ that does not contain a solution of $x + y = z$ (i.e., which is a sum free set)?
4. Colour the elements of $[1, \ldots, n]^r$ red and blue. How many monochromatic Schur triples are there?

Triangles in random graphs

Alex Scott

Let X be the number of triangles in a random graph $G(n, 1/2)$. Loebl, Matousek and Pangrac showed that X is close to uniformly distributed modulo q when $q = O(\log n)$ is prime. We extend this result considerably, and discuss further implications of our methods for the distribution of X. This is joint work with Atsushi Tateno (Oxford).

Combinatorial problems on the symmetric group: some applications of non-Abelian Fourier Analysis

David Ellis

We say a family of permutations $\mathcal{A} \subset S_n$ is *t-intersecting* if any two permutations in $\mathcal{A}$ agree in at least t places, i.e. for any $\sigma, \pi \in \mathcal{A}$, $|\{i \in [n] : \sigma(i) = \pi(i)\}| \geq t$. In 1977, Deza and Frankl proved that a 1-intersecting family has size at most $(n-1)!$, and conjectured that if n is sufficiently large depending on t, a t-intersecting family $\mathcal{A} \subset S_n$ has size at most $(n-t)!$. This was proved independently by the author and by Ehud Friedgut and Haran Pilpel in 2008, using the same techniques; we have recently submitted a joint paper. I will give a sketch of the proof, which uses an eigenvalue argument combined with representation theory of S_n.

We will then turn to the question of stability. In 2003, Cameron and Ku conjectured that for $n \geq 6$, a 1-intersecting family not contained within a coset of the stabilizer of a point cannot be larger than the family

$$\{\sigma \in S_n : \sigma(1) = 1, \sigma(i) = i \text{ for some } i > 2\} \cup \{(12)\}$$

which has size $(1 - 1/e + o(1))(n-1)!$, and that the extremal families are precisely the double cosets of this family. A generalization of this was proved by the author in 2008 using non-Abelian Fourier Analysis combined with a combinatorial argument: namely, for n sufficiently large depending on t, if $\mathcal{A} \subset S_n$ is a t-intersecting family which is not contained within a coset of the stabilizer of t points, then $\mathcal{A}$ cannot be larger than the family

$$\{\sigma : \sigma(i) = i \ \forall i \leq t,\ \sigma(j) = j \text{ for some } j > t+1\} \cup \{(1\ t+1), \ldots, (t\ t+1)\}$$

which has size $(1 - 1/e + o(1))(n-t)!$; the extremal families are precisely the double cosets of this family. This can be seen as an analogue for permutations of the Hilton-Milner theorem on non-trivial intersecting families of r-sets. I will sketch the main ideas of the proof in the $t = 1$ case.

If time permits, we will then discuss some related questions.

Some recent developments in bootstrap percolation

Rob Morris

Let G be a graph, r be an integer and $A \subset V(G)$ be a set of 'infected' vertices. The infection spreads according to the following rule: if a healthy site has at least r infected neighbours then it becomes infected, and infected sites are infected forever. Suppose the elements of A are chosen independently at random with probability p. At what point does percolation (infection of the entire vertex set) become likely?

I shall present some recent progress on this problem (known as bootstrap percolation) for the graph $G = [n]^d$, and also discuss several open questions. In particular, we still know very little about what happens when the number of dimensions $d = d(n) \to \infty$.

(This is mostly joint work with Jozsi Balogh and Béla Bollobás, and/or some subset of Daniel Ahlberg, Tom Coker, Hugo Duminil-Copin, Janko Gravner and Ander Holroyd.)

Arbitrary orientations of cycles in oriented graphs

Luke Kelly

An oriented graph is a simple graph in which each edge is assigned a direction. Sufficient conditions for Hamilton cycles in oriented graphs have been much studied in the last few years. This has included the confirmation of an old conjecture of Häggkvist by Keevash, Kühn and Osthus, who proved that every large oriented graph G with minimum in-degree and out-degree at least $(3|G| - 4)/8$ contains a Hamilton cycle. In this talk I will present a recent result showing that, up to a linear error term, this minimum in-degree and out-degree condition gives not just a Hamilton cycle but any orientation of a Hamilton cycle. This answers a conjecture from the 1997 paper of Häggkvist and Thomason in which they proved that a minimum in-degree and out-degree of $5|G|/12$ suffices and conjectured that $3|G|/8$ was the correct bound (both up to a linear error term $\epsilon|G|$). I will also discuss results and open problems for arbitrary orientations of short cycles, where we need different conditions for different orientations.

Hamilton cycles in random geometric graphs

Mark Walters

The Gilbert model of a random geometric graph is the following: place points at random in a (two-dimensional) square box and join two if they are within distance r of each other. For any standard graph property (e.g. connectedness) we can ask whether the graph is likely to have this property. If the property is monotone we can view the model as a process where we place our points and then increase r until the property appears.

In this talk we consider the property that the graph has a Hamilton cycle. It is obvious that a necessary condition for the existence of a Hamilton cycle is that the graph be 2-connected. We prove that, for asymptotically almost all collections of points, this is a sufficient condition: that is, the smallest r for which the graph has a Hamilton cycle is exactly the smallest r for which the graph is 2-connected.

This work is joint work with József Balogh and Béla Bollobás.

The early evolution of the H-free process

Peter Keevash

The H-free process, for some fixed graph H, is the random graph process defined by starting with an empty graph on n vertices and then adding edges one at a time, chosen uniformly at random subject to the constraint that no H subgraph is formed. Let G be the random maximal H-free graph obtained at the end of the process. When H is strictly 2-balanced, we show that for some $c > 0$, with high probability as $n \to \infty$, the minimum degree in G is at least $cn^{1-(v_H-2)/(e_H-1)}(\log n)^{1/(e_H-1)}$. This gives new lower bounds for the Turán numbers of certain bipartite graphs, such as the complete bipartite graphs $K_{r,r}$ with $r \geq 5$. When H is a complete graph K_s with $s \geq 5$ we show that for some $C > 0$, with high probability the independence number of G is at most $Cn^{2/(s+1)}(\log n)^{1-1/(e_H-1)}$. This gives new lower bounds for Ramsey numbers $R(s,t)$ for fixed $s \geq 5$ and t large. We also obtain new bounds for the independence number of G for other graphs H, including the case when H is a cycle. Our proofs use the differential equations method for random graph processes to analyse the evolution of the process, and also give further interesting information about the structure of the graphs obtained, including asymptotic formulae for a broad class of extension variables. This is joint work with Tom Bohman.

Random intersection graphs

Stefanie Gerke

A uniform random intersection graph $G(n,m,k)$ is a random graph constructed as follows. Label each of n nodes by a randomly chosen set of k distinct colours taken from some finite set of possible colours of size m. Nodes are joined by an edge if and only if some colour appears in both their labels. These graphs arise in the study of the security of wireless sensor networks. Such graphs arise in particular when modelling the network graph of the well known key predistribution technique due to Eschenauer and Gligor.

In this talk we discuss some basic properties of random intersection graphs, for example connectivity and the emergence of the giant component. This is joint wok with Simon Blackburn and Paul Balister.

Synchrony and Asynchrony in Neural Networks

Angelika Steger

In this paper we pick up on a model described recently by DeVille and Peskin (Bulletin of Mathematical Biology, to appear) for a stochastic pulsecoupled neural network. The key feature and novelty in their approach is that they describe the interactions of a neuronal system as a discrete-state stochastic dynamical network. They show (experimentally and by some estimates in an associated mean-field limit of the model) that their network can exhibit both synchronous and asynchronous behavior. They also exhibit a range of parameters for which the network switches seemingly spontaneously between synchrony and asynchrony. In synchronous behavior the firing of one neuron leads to the firing of other neurons, which in turn may set off a chain reaction that often involves a substantial proportion of the neurons. There are strong analogies to the giant component phenomenon in random graph theory.

In this talk we present a rigorous analysis of the model of DeVille and Peskin, thereby answering in particular their questions about the actual parameter settings resp. thresholds for which these changes between synchronous and asynchronous behavior occur. We also provide insights into the coexistence of synchronous and asynchronous behavior and the conditions that trigger a "spontaneous" transition from one state to another.

Joint work with Fabian Kuhn, Konstantinos Pangiotou, and Joel Spencer

A Complexity Dichotomy for Hypergraph Partition Functions

Leslie Ann Goldberg

Abstract: The talk will introduce "partition functions", which arise in many computational contexts, and will discuss the complexity of computing them. It will explain what a "dichotomy theorem" is, and why we want such theorems (essentially, we want them so that we can better understand the boundary between the class of easy-to-compute functions and the class of functions that cannot be efficiently computed).

The particular technical problem which forms the foundation for the talk is the complexity of counting homomorphisms from an r-uniform hypergraph G to a symmetric r-ary relation H. We give a dichotomy theorem for $r > 2$, showing for which H this problem is easy to compute and for which H it is #P-complete. Our dichotomy theorem extends to the case in which the relation H is weighted, and the partition function to be computed is the sum of the weights of the homomorphisms. This problem is motivated by statistical physics, where it arises as computing the partition function for particle models in which certain combinations of r sites interact symmetrically.

(joint work with Martin Dyer and Mark Jerrum)

Wiretapping: the nucleolus of connectivity

Rahul Savani

We consider the problem of maximizing the probability of hitting a strategically chosen hidden network by placing a wiretap on a single link of a communication network. This can be seen as a two-player win-lose (zero-sum) game that we call the wiretap game. We also study this problem in a cooperative game setting. For the cooperative game we study, the connectivity game, we provide a polynomial-time algorithm for computing the nucleolus. The nucleolus of the connectivity game corresponds to a maxmin strategy in the wiretap game with desirable properties, for example, it minimizes the number of pure best responses of the player choosing the hidden network. The nucleolus of a connectivity game is also of interest as a mechanism for cost-sharing, where those links most likely to be used in connected spanning subgraphs are seen as most important or, alternatively, as most vulnerable. Joint work with Haris Aziz, Oded Lachish, and Mike Paterson

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"The Norman Biggs Lecture"

On the Category of Graphs

Jaroslav Nešetřil

We present a survey of categorical aspects of graph homomorphisms: constructions and global properties centered around notions of universality and density.

THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

THURSDAY 21st MAY 2009

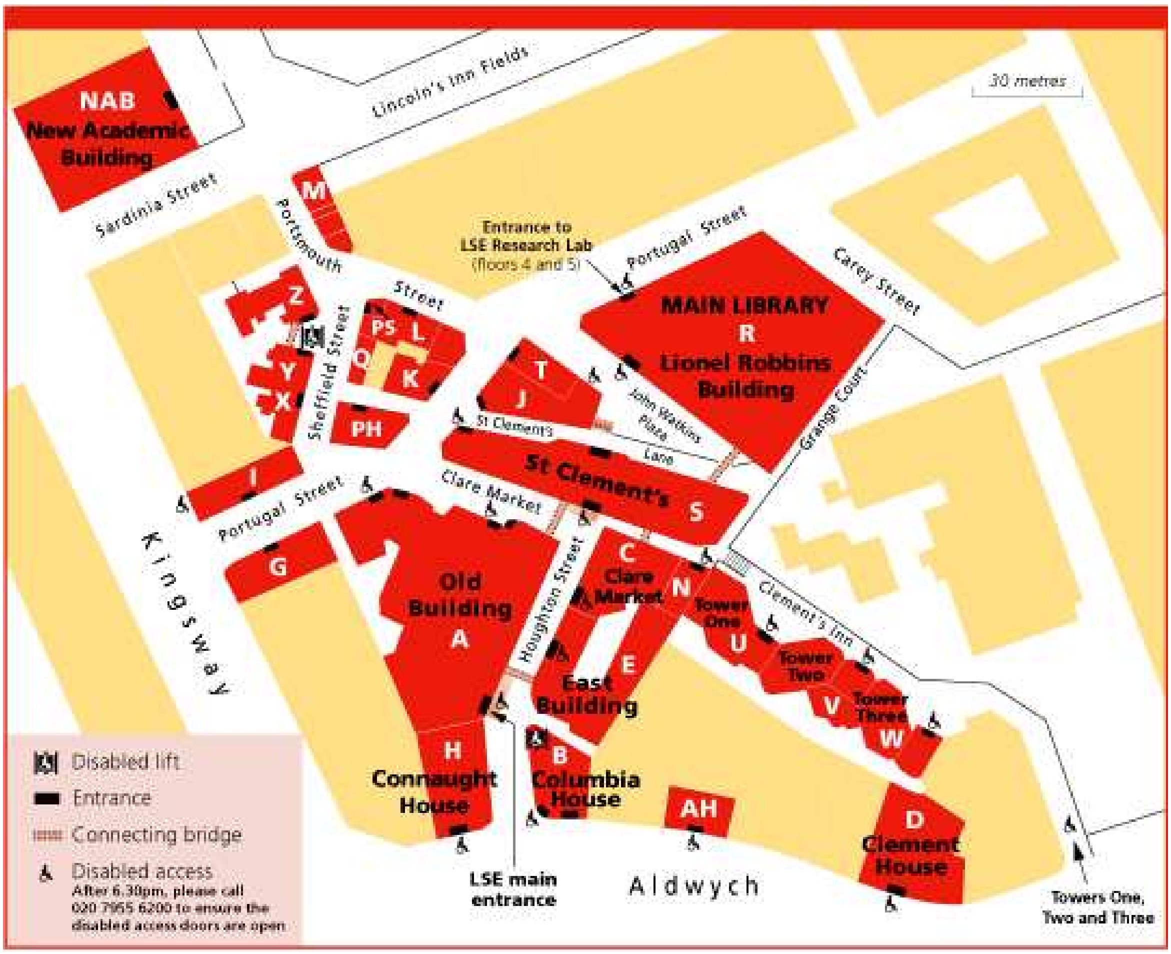
The second of the two linked colloquia in combinatorics will be held at the London School of Economics on Thursday 21st May, starting at 10.00am. Everyone interested is welcome to attend any part of the event. The talks will be held in the New Theatre (E171), in the East Building.

10.00 - 10.45	**P. Keevash** (Queen Mary) *The early evolution of the H-free process*
10.45 - 11.05	***Coffee break*** *- E168*
11.05 - 11.50	**S. Gerke** (Royal Holloway) *Random intersection graphs*
11.55 - 12.40	**A. Steger** (ETH Zurich) *Synchrony and asynchrony in neural networks*
12.40 - 14.05	***Lunch*** *(own arrangements - options on campus and nearby)*
14.05 - 14.50	**L. Goldberg** (Liverpool) *A complexity dichotomy for hypergraph partition functions*
14.50 - 15.10	***Coffee break*** *– E168*
15.10 - 15.55	**R. Savani** (Warwick) *Wiretapping: the nucleolus of connectivity*
16.00 - 17.00	"The Norman Biggs Lecture" **J. Nešetřil** (Prague) *On the category of graphs*

Support for this event by the London Mathematical Society, the British Combinatorial Committee and an anonymous donor, is gratefully acknowledged.

Finding Your Way Around The LSE

The number indicates both the floor and the room. Room numbers in the basement begin with a zero, numbers 1-99 are on the ground floor, 100-199 are on the first floor, 200-299 on the second floor and so on. Some rooms are identified by name rather than number.



A – Old Building	N – The Anchorage
AH – Aldwych House	NAB – New Academic Building
B – Columbia House	PH – Parish Hall
C – Clare Market Building	PS – Portsmouth Street
D – Clement House	Q – Sheffield Street
E – East Building	R – Lionel Robbins Building, LSE Library
G – 20 Kingsway	S – St Clement's Building
H – Connaught House	T – The Lakatos Building
I – Peacock Theatre	U , V, W – Tower One, Two, Three
J – Cowdray House	X – St Philips Building – Health Centre
K – King's Chambers	Y – St Philips Building – South Block
L – Lincoln Chambers	Z – St Philips Building – North Block
M – 50 Lincoln's Inn Fields	